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Two-Dimensional Packing Problems in Telecommunications

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I thank IFORS and SOBRAPo for the honor of this invitation. I will present a concise description of the development of an interdisciplinary research applied to real world problems, jointly developed by four teams, in chronological order:

Nokia Siemens laboratories: research group on the IEEE 802.16/WiMAX standard;
University of Pisa: research group on Computer Networking (Luciano Lenzini);
University of Bologna: research group on Combinatorial Optimization (S.M.)
Technical University of Eindhoven: research group on Theoretical Combinatorial Optimization (Gerhard J. Woeginger).

The project has been developed following the classical steps of an applied research:

1. birth from a real world problem;
2. development of mathematical models for its combinatorial aspects;
3. theoretical analysis;
4. definition of mathematical models for the real world problem;
5. evaluation of the technological constraints;
6. development of solution algorithms;
7. implementation and experimental evaluation on realistic scenarios.

A synthetic overview of these steps is presented in the following. A more complete description can be found in [1].

The birth: an optimization problem in telecommunications. In telecommunication systems adopting the IEEE 802.16/WiMAX standard, a fixed station transmits and receives data packets to and from other stations (e.g., our mobile phones), and all transmissions are performed using (time × frequency) rectangular frames, called downlink zones, where the packets are stored as rectangles.

The fixed station must maximize the frame utilization by deciding which packets will be included in the next transmission phase, arranging each selected packet into one or more rectangular regions, and allocating the resulting regions to the frame without overlapping.

The models: new two-dimensional packing problems. In a standard two-dimensional bin packing problem one has to allocate, without overlapping, a given set of rectangles to the minimum number of identical large rectangles of prefixed width and height. In the considered real world problem the items to be allocated are instead data packets. The \( j \)-th data packet is an amount of information, in practice a number, that may be interpreted as an area of size, say, \( a_j \). Such area must be arranged as a \( w_j \times h_j \) rectangle such that \( w_j h_j \geq a_j \) (or as a number \( mj \) of rectangles, called sub-areas, such that \( w_{j1} h_{j1} + \cdots + w_{jm} h_{jm} \geq a_j \)). The resulting rectangle(s) must then be optimally allocated to the downlink zone. In addition, each created and allocated rectangle needs information (height, width, coordinates), that has to be included in the downlink zone, i.e., a portion of the zone, proportional to the number of rectangles it contains, is used for the so-called maps transmission.

Theoretical analysis: computational complexity and approximability. When a new optimization problem arises, it is advisable to preliminarily answer some questions. How difficult is the problem? Can it be solved in polynomial time? If not, can it be approximated with some worst-case (or asymptotic, or probabilistic) guarantee in polynomial time? Can it be solved efficiently in practice?

To answer these questions, let us consider the simplest combinatorial optimization problem we can “extract” from the industrial problem:

Area Packing: given \( n \) areas, and a single rectangle (\( \text{bin} \)), is it possible to arrange each area as a rectangle in such a way that all resulting rectangles can be allocated to the bin without overlapping?

A simple transformation from a variant of Partition shows that this problem is ordinary NP-complete. Sophisticated techniques, using tools from number theory and transformation from a variant of Three-Partition, prove that it is strongly NP-complete. Hence the problem cannot be solved in polynomial time, nor in pseudo-polynomial time, unless \( P = NP \). However, its optimization version can be approximated with worst-case performance guarantee in polynomial time as follows.

As we have seen, in the real world problem the size of the downlink zone that is wasted for maps transmission is proportional to the number of created sub-areas. It is then reasonable to assume that any area \( a_j \) can be arbitrarily split into any number of integer rectangular sub-areas (at most \( a_j \) unit squares), and to ask for packing all areas, without overlapping, into a given bin (of size no less than the sum of all areas) by minimizing the number of created rectangular sub-areas. A 3-approximation algorithm has been proposed in [2], where it is proved that it provides, in linear time, a solution that uses at most three times the minimum number of sub-areas. Such worst-case is tight.

The real world problem: four technological variants. There are three main differences between the theoretical problems outlined above and the telecommunication problems at hand:

- • the areas (packets) cannot be split in an arbitrary way: for each of them, a list of the feasible sub-areas into which it can be split is provided as part of the input. For each area we must define one or more rectangular regions containing the sub-areas. Note that this can make it impossible to completely pack all areas;
- • each sub-area has a profit (corresponding to its transmission priority), and the objective function is to maximize the total profit of the packed areas;
- • as already mentioned, the mapping of the packing must be stored in the frame, and minimizing the number of rectangles leads to minimizing the size of the map. However, as an additional difficulty, the actual size of the map can only be computed once the packing has been decided. =>

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In addition, it was requested to evaluate different technological characterizations, corresponding to different ways in which the downlink zone can be implemented. The first two options (problems P1 and P2 in the following) had a general rectangular downlink zone, with two different ways of storing the map. Two additional options (problems P3 and P4 in the following) had a more rigid way of allocating the areas.

Evaluation of the technological constraints. Independently of the technological variant, the planned system had to be implemented using sets of standard PCs. The technological constraints were extremely tough:

- each PC must perform 500 transmissions per second, i.e.,
- every 2 milliseconds it is necessary to read the input data, execute the optimization algorithm, produce the output (packing and map), and transmit the corresponding packets; however,
- each transmission takes 1 millisecond, i.e.,
- each instance must be completely solved within 1 millisecond!

Although real world instances are relatively “small” (they include few tens of packets) this requirement was really tough.

Development of solution algorithms. For problems P1 and P2, a recursive algorithm was implemented (in two versions). The algorithm is based on the alternate execution of two very fast heuristics, one of which is the generalized assignment problem. For problems P3 and P4, the particular structure imposed to the downlink zone allowed to conveniently adapt a classical heuristic algorithm (see [3]) for the generalized assignment problem.

Implementation and experimental evaluation on realistic scenarios. All algorithms (see [4], [5] for detailed descriptions) were implemented in C and tested on a simulator that generated (for each of the four technological variants) tens of thousands of instances representing different scenarios of transmission. The experiments were performed on a 1.66 MHz Pentium M Centrino laptop running Cygwin, and the results were extremely satisfactory. For all instances the proposed algorithms produced, within the 1 millisecond time limit, solutions of value very close to the theoretical optimum.

Bibliography

Getting to Know: The EURO Working Groups
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For groups of researchers interested in a specific Operational Research topic, EURO provides an organizational framework in the form of EURO Working Groups (EWGs). A very important part of EURO, EWGs provide a forum for promoting research in various OR areas.

History
Several EURO Working Groups were established at EURO’s very first conference in 1975. Since then, many have been put up and grew, while others have been disbanded. An account of the early history (up to 1983) can be found in J. Krarup, Profiles of the European Working Groups, EJOR 15 (1984) 13-37.

Since 1993, the liaison officer between EURO and the Working Groups is the Vice President 2. The EURO website (http://www.euro-online.org) EURO working group section reports on the activities of the each group. Currently, EURO hosts 29 active Groups, with the last one established January this year. This number indicates the success that EURO has achieved with this instrument. In turn, EWGs have contributed to the increasing success of EURO and IFORS conferences.

EURO’s Commitment to the EWGs
EURO provides organizational and financial support to the EWGs. The amount of financial support is based on previous years’ activity indicators (e.g. number of sessions organized in EURO K and IFORS conferences, publication of special issues in highly regarded scientific journals, number of participants in EWGs meetings) and on activities planned (e.g. EWGs meetings, organization of summer and winter institutes).

Funding can be used to cover:
- Current administrative costs related with the management and running of the EWG, including printing costs of common stationery and setting up/maintaining a group - specific webpage.
- Costs of the EWG’s specific meetings, organized outside the major OR Conferences, in particular travel and accommodation costs of invited guest speakers (not members of the EWG), EWG members from weak currency countries and early stage OR researchers (actual Ph.D. students and post-doctoral researchers for a period not exceeding two years after their Ph.D. defence).
- Other costs related with the dissemination and development of the EWG’s area, e.g., costs of initiating and/or publishing results of EWG activities or costs for invited survey contributions on specific EWG related topics to be submitted for publication in EURO journals.

The association with EURO enables the EWGs to create partnerships with Companies and other organizations with with whom research projects are launched and from whom additional funding may be sourced.

Benefits of Membership
Membership is open to individual members of any EURO member society and to persons who are not members of a EURO national OR society. >>