Invited Review

Use of location-allocation models in health service development planning in developing nations

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Abstract

There is considerable evidence that because of poor geographical accessibility, basic health care does not reach the majority of the population in developing nations. Despite the view that mathematical methods of locational analysis are too sophisticated for use in many of these nations, several studies have demonstrated the usefulness of such methods in the locational decision-making process. This paper reviews the use of location-allocation models in health service development planning in the developing nations. The purpose of this review is to examine the suitability of these methods for designing health care systems and their relevance to overall development problems in such countries. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The role of location analysis in planning services for regional development is well known. One of the tools for such analysis is quantitative location-allocation modeling. It provides a framework for investigating service accessibility problems, comparing the quality (in terms of efficiency) of previous locational decisions, and generating alternatives either to suggest more efficient service systems or to improve existing systems. In the context of developing nations, locational decisions are generally taken locally by government officers or by local elected leaders or by both. In the absence of any formal analysis and generation of alternatives, the final decision may be made on political or pragmatic considerations. As a result the decisions can very often be far from optimal [24,57]. Despite the view that mathematical methods of locational analysis are too sophisticated for use in most developing countries, many studies have demonstrated the usefulness of such methods in the locational decision-making process. In this paper we review a number of location-allocation studies that have been undertaken for health service development planning in developing nations.

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There are two objectives of this review paper. First, we examine the role of location-allocation models in planning health care systems and second, consider their relevance to development problems in developing nations. This paper is divided into four sections. In Section 2 the relationship between health and economic development is discussed. In Section 3, public facilities, the structure of the health care system and location-allocation models are discussed and we develop a framework for reviewing the studies. The actual review is given in Section 4 and the paper is concluded in Section 5 with a discussion on future research directions.

2. Health and economic development

Despite remarkable reductions in the incidence of disease and mortality rates over the last four decades, the degree of ill health in developing nations remains enormous [81]. Available data on health indicate that extremely high levels of preventable illness prevail in these countries. For instance, at least five million children are blinded or crippled each year, most of them as a result of polio, a disease for which effective vaccines are available. Infant mortality in many developing countries is extremely high. Therefore, further improvement in health will largely depend on the capacity of health systems to deliver primary health care (PHC) services. This view has been widely accepted and well documented. The international conference on primary health care at Alma-Ata ([1], p. 7) declared that “Primary health care is the key to achieving an acceptable level of health ... in the foreseeable future and in the spirit of social justice. It is equally true for all countries ... For developing countries in particular, it is a burning necessary”. Gradually extending the PHC services to the rural people of developing nations, the following objectives could be achieved [37].

- To increase equity in the distribution of health benefits and reduce morbidity and mortality.
- To reduce population growth rates in the longer run.
- To stimulate economic growth through a healthier population.

There is evidence to believe that the development of health is essential for economic development. For instance, a study in Malaysia demonstrated that reduction of malaria in a rubber estate caused output per worker to rise 17-fold [5]. It is widely acknowledged that it is no longer tenable to draw a distinction between economic and health development. The development of health along with other social improvements is necessary to achieve economic goals. Correspondingly, economic development is necessary to achieve most of the social goals. The importance of health and its relation to development in developing nations can be observed in a statement of a former President of the Institute of the US National Academy of Sciences. He said that “Diseases in developing countries take such a terrible toll in human suffering and economic loss that they are at the heart of the whole problem of development ... The developing countries may become the never-to-be-developed countries unless the burden of illness can be greatly eased” [39]. Evidently, development in health is a precondition for economic development of a nation. In a simple example of the economic benefits of health provision, Dinwiddy and Teal ([17], p. 216) write: “immunisation programmes for contagious diseases ... benefit far more people than the number actually receiving the treatment”.

3. Public facilities, health delivery systems, and location-allocation models

There are two principal categories of central location problems, whose objectives are often different. Trying to find the best locations for public facilities, such as schools, parks, utilities, sports and health centres is a different problem from that of locating private facilities, such as banks, shops and private leisure facilities. (This paper is only concerned with the location of central facilities. Hodgart [31] has classified location problems into three groups, those locating central facilities, semi-desirable facilities and obnoxious facilities.) Central facilities are facilities to which people must travel to receive the service, or from which a service is provided to the whole community of inter-
est. It is generally felt that the closer (usually measured in terms of distance or time) the facilities are to the users, the better the service provided.

Research for locating health facilities in the context of the problems of developing nations has developed two categories of models. Some research has been directed towards the location of components of a health care system in which facilities are considered to be of one type (with respect to the level of service provided). We refer to these models as single-level location-allocation models (SLAMs). Patel [54] and Berghmans et al. [7] give some examples. However, it is widely recognised that most health care systems in developing countries are organised as hierarchical systems, just as they are in industrialised nations. A patient in a typical developed country goes to see a doctor or nurse, relatively close to his or her home. If necessary, the patient attends a hospital which provides facilities which are not available from the doctor. Some patients will then progress to a third level of hierarchy, a specialist hospital. The same is true in developing countries. For example, the primary health care delivery system in Bangladesh consists of Community Clinics (CCs), Health and Family Welfare Centres (HFWC); and Thana Health Complexes (THCs). In this system, Community Clinics are considered to be the first point of contact between rural people and the health system. A patient whose medical problems cannot be dealt with at such a clinic may be referred to a HFWC or THC, and a patient who goes to a THC may be referred to a HFWC. The system consisting of these three distinct types of facilities may be termed a “3-hierarchical system”. Generalising this notion, it may be stated that a health care system that consists of \( f \) (\( \geq 2 \)) distinct types of facilities which collectively deliver services can be called an \( f \)-hierarchical system. Narula [49] and Tien et al. [75] give detailed descriptions of these hierarchies, and the interested reader should refer to these papers. Addressing the location problem on a regional level, some researchers have considered the problem as a hierarchical system. We call these hierarchical location-allocation models (HLAM). The studies by Banerji and Fisher [3] and Harvey et al. [29] are some examples of HLAMs.

A location-allocation model is a method for finding optimal sites for facility locations. The method involves simultaneously selecting a set of locations for facilities and assigning spatially distributed sets of demands to these facilities to optimise some specified measurable criterion. The most important issue raised in the process of solving location problems is the selection of a suitable criterion or objective function. The formulation of an objective function depends greatly on the ownership of the organisation, both whether public or private and the nature of the facilities, as has been mentioned earlier. For instance, private sector facilities are often located to fulfil precisely stated objectives, such as to minimise cost or to maximise profit. In contrast, the goals and objectives of public facility location are more difficult to capture [63,67]. For example, if the problem is to locate emergency ambulance services, a possible criterion would be to minimise the average distance (or time) an ambulance must travel in order to reach a random incident. Another appropriate criterion could be to minimise the maximum distance that the ambulance must travel to reach an accident. (An advertisement for IBM in 1978 drew attention to a planning decision made in the 19th century: “Horses pulling a fire wagon, it was once decided, can run no further than 10 city blocks. For that reason, fire stations in many American cities were located 20 blocks apart.” [42].) There have been comprehensive reviews of the general problem of locating emergency facilities (not simply in developing countries) in [64,65].

Different interpretations of the goal of maximum public welfare lead to a number of possible location-allocation problems.

One of the most popular models for public facility location problems is the \( p \)-median problem [28,62]. (It has also been useful in planning control of disease [73].) The problem can be defined in the following manner: given discrete demands locate a number \( (p\text{ or less}) \) of facilities so that the total weighted travel distance or time between facilities and demand centres is minimised.

The \( p \)-median problem is attractive, since the smaller the total (or average) weighted travel distance (time), the more convenient for the
users to get to the nearest facility. It is assumed
that all users of the facility choose to travel to
the closest one. However, this model does not
take the ‘worst case’ situations into account and
so it may result in solutions which are not ac-
ceptable from a service point of view. Solutions
which minimise the weighted travel distance may
be inequitable, forcing a few users to travel far.
This may mean that the remotest users do not
actually travel to their nearest facility, or to any
facility. It has frequently been observed that the
usage of service facilities declines rapidly when
the travel time (distance) exceeds some critical
value. This is a typical situation with the use of
rural health facilities in developing nations
[58,72]. Therefore it is quite reasonable to con-
sider maximum distance or time constraints in
formulating a location problem. This leads to
the formulation of the location set covering
problem (LSCP) [78]. Here the problem can be
defined as: find the minimum number of facilities
and their locations such that each and every
demand centre is covered by at least one facility
within a given maximal service distance (time).
This formulation has been used in a developed
country for locating kidney dialysis machines, a
form of treatment for which the patient must
make frequent, repeated journeys [22]. A related
problem, known as the \( p \)-median problem, is
concerned with finding an efficient set of facility
locations which can be associated with districting
the catchment areas for two or more levels of
facility. Some heuristic approaches to it are de-
scribed in [70] and an exact algorithm is found
in [71].

In reality there may not be enough resources to
provide the number of facilities which would be
required by the optimum solution to this problem.
This is particularly the case in developing nations.
Then, the decision maker may abandon the goal of
total coverage and attempt instead to locate the
facilities in such a way that as few people as pos-
sible lie outside the desired service distance. This
means the problem is to maximise coverage within
a desired service distance by locating a fixed
number of facilities. This problem is referred to as
the maximal covering location problem (MCLP)
[12].

The mathematical formulations of these basic
problems are well documented in the literature,
and will not be repeated here. Over the last two
decades many variants of these models have been
developed ([9,14,38,42,79]). Some of these variants
allow for multiple or sequential objectives (for
instance [21]). This paper reviews studies which
have addressed the health facility location prob-
lem in developing nations. However, the discus-
sion is limited to the network forms of the
\( p \)-median and covering problems and their vari-
ants. For models addressing the same problems on
a plane, see [18,19] whose models are \( p \)-median
HLAMs dealing with health facility location in
Turkey. Some researchers have used other meth-
ods to design health systems in developing coun-
tries. For instance, Bhatnagar [8] showed how
interactive graphics could be successfully applied
in locating service centres in rural India. Hodgson
[34] applied a model in the health care system in
rural India which is based on a negative expo-
nential adoption of Reilly’s retail gravitational
law. This reflects the unwillingness of patients to
make a journey greater than a certain distance or
time. The model, however, is both data intensive
and data sensitive. This casts serious doubts about
its applicability in a developing nation. More re-
cently, Massam and Malezenski [43], and Datta
and Bandyopadhyay [15] demonstrated the role of
decision support systems (DSSs) on health plan-
ing and location in Zambia and India, respec-
tively. This review of the use location-allocation
models in developing nations follows the frame-
work shown in Fig. 1.

4. Review of location-allocation studies

In this section we review several health facility
location-allocation studies conducted in the con-
text of developing nations. These studies were de-
sign to:
- Find a set of optimal sites.
- Locate optimal sites in a new area.
- Measure the effectiveness of past location deci-
sions.
- Improve existing location patterns.
4.1. Finding optimal site(s)

Perhaps the earliest location-allocation study which explicitly addressed the problem of locating health facilities in a developing country is one by Gould and Lienbach [27]. In this study, the problem of locating hospitals and determining their capacities (in terms of number of beds) in the western part of Guatemala was considered as a \( p \)-median problem on the existing road network. The transportation algorithm was used to solve the problem [16]. The population centres were treated as surplus nodes while the hospital capacities were defined as quantities to be filled. The transportation costs from a population centre to the hospital were calculated considering the most efficient service route available. However, no mathematical formulation of the problem was given. First, the model was solved to locate three regional hospitals for 18 population centres. The optimal set of hospitals produced the shortest total travel distance. However, there were some awkward flow patterns between communities and hospitals. It appeared that some communities were not assigned to the closest hospital, because these were all of the same size. Changing hospital capacities eliminated this unacceptable flow. Apart from finding the size and sites of hospitals based on the existing road networks, the study made useful suggestions relating to the impact of a change in the transportation network on the location of centres and the assignment of communities to them. Such comments may be seen as simple sensitivity analysis, and as pointers to the answers to “What if . . .?” questions.

A similar type of study was conducted to locate rural health clinics in the Eastern Region of Upper Volta by Mehretu et al. in 1983 [45]. The objective was to locate clinics such that the total weighted travel distance between clinics and villages was minimised subject to the constraint that no one would travel more than a maximum distance of 5 km. Their problem was a modified \( p \)-median problem, defined as the \( p \)-median problem with maximum distance constraints [38,78]. First, 635 villages in the study area were arbitrarily grouped into 94 village clusters referred to as programming units. Then the facilities were located in each programming unit separately (a geographically constrained problem) using the Teitz and Bart [74] algorithm. In all, 222 health clinics were necessary to cover all the villages. However, one is left with the impression that the same objective could have been achieved using fewer facilities if only one problem were to be solved for the entire region. It is a common feature of models whose objective is minimum cost, not simply location models, that extra constraints increase the cost; so restricting the solutions, programming unit by programming unit, will

![Framework for the review](image-url)
almost certainly give a higher cost and more facilities than a less constrained problem.

The study by Patel [54] concentrated on the choice of locations for service centres in rural India. The list of centres included public health centres, primary schools, agricultural extension offices, post offices and fair-price shops. These were considered to be essential to provide the minimal infrastructure, education and health facilities for rural people. 44 out of 237 villages were chosen as the potential sites for service centres. The criteria used included population, growth potential, proximity to highways, the nature of adjoining areas and the existence of a village market. Then the problem was formulated as a constrained minimisation problem. No service centre had to be greater than a predetermined ‘maximum distance’ from the community that is served. There was a budget constraint, which made the minimisation of ‘maximum distance’ a difficult integer nonlinear programming problem. (The nonlinearity was essentially combinatorial, and could have been removed at the cost of many extra variables and constraints.) For fixed distance, the cost could be minimised as (effectively) a dual problem, using an algorithm similar to one used by Toregas et al. [78] for locating emergency service facilities. Part of Patel’s paper deals with the problems of including new transport infrastructure, as well as the resources for health provision. The method of Lemke et al. [40] was used to solve this for eight values of maximum distance, yielding an empirical plot of cost versus maximum distance. It was shown that despite a cut in the original budget of 75%, the maximum distance had to be increased by only 20%, from five miles to six miles. Patel’s analysis showed that the original budget for the project was far too large for the basic location problem. Therefore, with optimal location, the service level of five miles could have been obtained with a cost of only 42% of the original budget. There was no feasible solution for smaller service levels.

Eaton et al. [21] in their study described the development, use and implementation of location-allocation techniques for ambulance deployment in Santo Domingo, capital of the Dominican Republic. Here the problem was considered as an LSCP with back-up cover which sought to maximise the multiple coverage of demand within a specified response time, with the minimum number of ambulances. The problem was solved (by hand, because computers were not available!) for 214 demand areas in Santo Domingo using several maximal response times. The study suggested that between 8 and 23 vehicles would be necessary to cover all demands within 5–10 minutes. The study achieved two main benefits. First, it provided the Ministry of Health with a basis for establishing an emergency medical service system in Santo Domingo. Second, the confidence in Operational Research techniques was enhanced by the leading role of Dominican participants in this study. Reid et al. [61] used a similar model to find locations of medical supply centres in Ecuador.

Bennett et al. [6] demonstrated how the use of location analysis could be successfully integrated into health centre planning in rural Colombia. In the late 1970s the local health planners of the state of Valle del Cauca, Colombia initiated a study to determine rural health centres from which to recruit rural health workers (promontoras) and at which to base ambulances. They considered several factors such as the availability of water and electricity in a village, and the maximum travel distance from health centre to any village while determining health centres. Based upon a detailed data survey and using their intuitive judgement local health planners determined 24 health centres which were accessible to 78% of the population. The authors worked in consultation with the local health planners. Their paper formulated the problem as an MCLP which was solved by a ‘greedy adding with substitution’ (GAS) heuristic and tested for optimality by linear programming. The results from the GAS heuristic showed that it required only 15 centres to cover the same percentage of the population. With 24 optimally located health centres, the same number as proposed by the local planners, 90% of the population could be covered. The study illustrated that location analysis could be combined with the intuitive judgement of the planner in developing countries.

Oppong [52] studied the effect of changes to communication links due to the rainy season in a
tropical country. This paper was concerned with the location of health facilities in Suhum District, Ghana. The problem was formulated both as a $p$-median problem and as an MCLP. The author developed a number of scenarios for the decision-makers to consider. The use of such scenarios opens the way to multi-criteria decision analysis, with trade-offs between several performance measures. However, little mention is made of the possibility of formulating the problem in this way, which would seem to be a more appropriate approach.

Rahman and Smith [56] conducted a study to find suitable sites for new health facilities with respect to the existing facilities to improve the accessibility of the overall health system to people in Tangail Thana (Sub-district), Bangladesh. The study region has an area of 210 square kilometres and consists of 8 unions and a municipality. (A union is the lowest administrative zone in Bangladesh.) According to the Government of Bangladesh’s plan, 30 Community Clinics were to be located to serve about 240,000 people living in 212 villages. Those villages, without a health facility, which have a population of 500 or more, and have potable water and electricity were considered to be the potential sites for Community Clinics. The location-allocation problem was then considered in two ways. First, locating the facilities taking one union at a time (similar to Mehretu et al. [45]). These problems are referred to as geographically constrained problems. Second, locating all facilities at a time by solving one global problem (a geographically unconstrained problem). Community Clinics were located solving an MCLP and four sets of location sites were suggested. Generally, the solutions of the geographically unconstrained problem were found to be more efficient than the solutions of the constrained problems. The study suggested that implementation of one of the solutions of the unconstrained problem would make the health delivery system 60% less costly, while serving the entire population with a maximum travel distance of 2 km. The objective of the study however, was not to suggest a single optimal decision, rather to develop and test feasible decision processes in the light of the Government’s health plan.

The studies above focused only on the accessibility component of the health care coverage problem. Tien and El-Tell [76] proposed a $p$-median model which simultaneously addressed two components, accessibility and availability. Availability as a measure has been used in health service problems in at least two senses. In developed countries, it frequently refers to the ratio of health service professionals to a given population. In the paper availability was measured in terms of the time for which a physician was available in health centres and village clinics. These professionals were only available for part of the working week, and divided their time between the clinics. The model was applied to the Mafraq district of Jordan. The problem was to locate village clinics and health centres and to identify a relationship between them. This relationship was based on the organisational attachment of one or more (village) clinics to every health centre and on the presence of a health centre-based physician at each clinic. It was necessary to locate the health centres at the most populated villages having the essential support services (electricity, water, telephone and good road transport). The problem was formulated as a zero–one integer programming problem, based on Revelle and Swain’s formulation [62] with extra constraints which used the availability measure. It was solved on the MPSX package as a relaxed linear programme. The results demonstrated that by both reallocating the villages to the clinics and the clinics to the existing health centres considerable improvement (in terms of coverage) could be made.

The above studies focused on locating components of health care system in which facilities are considered to be one type. Thus, they belong to the class of models which has been identified as single-level location-allocation models (SLAMs). The model by Tien and El-Tell [76] which claimed to be a quasihierarchical location-allocation model, is essentially a location routing problem of the general nature described by Narula [50]. Banerji and Fisher [3] described the application of hierarchical location analysis for integrated area planning in Andhra Pradesh, India. The problem of locating health facilities in rural villages is a part of the larger study. Here, a successively inclusive facility
hierarchy was assumed. The proposed formulation was based on a combination of the LSCP and the $p$-median problem. However, no mathematical formulation was given. Later, Weaver [80] gave a mathematical formulation of their problem. First the LSCP was solved to determine the number of facilities required at each level, for a given maximum allowable distance at each level of hierarchy. Then, given the number of facilities at each level, the $p$-median problem was solved to determine the optimal locations of the facilities. The procedure involved here was first to locate the highest-level facility and then to move down the hierarchy. This strategy of successively locating some higher to lower level facilities is termed the top-down approach [35]. The LSCP and $p$-median problem was solved using the Banerji [4] heuristic and the Teitz and Bart [74] heuristic respectively. The health care delivery system described here is an example of a $2/I/U$ location-allocation model. The notation for the models is explained in detail in Appendix A.

Like Banerji and Fisher [3], both the studies by Harvey et al. [29] and Moore and ReVelle [47] considered the problem of locating facilities as hierarchical systems. Their subject of the first of these was the identification of nodal hierarchies of growth centres in Sierra Leone. Each centre provides 35 different functions and delivering health care was one such. A successively inclusive facility hierarchy was assumed. Repeated use of a $p$-median algorithm proposed by Hung and Brown [36] gave a solution to the problem. Unlike Banerji and Fisher [3] a bottom-up approach was used for locating facilities. This means, first the problem was solve to locate the lowest-level facility and then moved up the hierarchy to locate the higher-level facilities. In this study the problem of choosing five different types of growth centres is an example of $5/I/U$ location-allocation model.

Moore and ReVelle [47] extended the MCLP and applied it to a two-tier hierarchical health care delivery system in Honduras. The problem was to simultaneously locate a fixed number of clinics and hospitals. To maximise the population with clinic services available within a distance standard set for clinics, and with hospital services available within a hospital distance standard. Out of 144 demand nodes, 62 nodes (those with over 50 people) were chosen as the feasible facility sites. Of these only 28 (those with over 100 people) were considered feasible for the higher level facilities (hospitals). The problem was solved using a commercially available linear programming package (MPSX). The results were represented as a curve of population coverage versus the investment in facilities, instead of the number of facilities at each level. This was done in order to present the results in a two dimensional manner (coverage and investment). The hierarchy of the health system was considered to be successively inclusive. The problem can be categorized as a $2/I/I/U$ location-allocation model.

### 4.2. Locating facilities in a new area

Berghmans et al. [7] reported on a study which dealt with a problem of locating health centres in a completely new city (Yanbu al Sinaya) in Saudi Arabia. Here the health centres were considered to be the first point of contact between the population and the health system. Taking into consideration four quantitative criteria the problem was formulated as a LSCP. These were the ratio of doctors to population, the maximum service distance $S$, the equity in service (defined by insisting “that all health centres should be similar in staff and equipment”) and the number of doctors per centre. First, the projected population of the city was distributed between 36 regions, approximately equally. The city was then described by a graph with 36 vertices of equal weight, each vertex representing the ‘central point’ of a region. Two vertices were joined by a link if they corresponded to two adjacent regions. The problem consisted of finding the number of centres and their location anywhere on the links of the graph so that all the vertices were contained within the maximum service distance $S$. The problem was solved using the Garfinkel and Nemhauser [25] heuristic for different values of $S$. Because the centres did not need to be located at vertices, the solution space was effectively infinite, and heuristic approaches essential. The results were compared with solutions of the problem which assumed that all centres would be restricted to the existing vertices. In general, the
study provided the local health consultants with a cost/benefit analysis using accessibility to the health care delivery system (S) as the main parameter.

4.3. Measuring the effectiveness of past locational decisions

There are studies which have explicitly addressed the efficiency of earlier locational decisions. Among these are those by Rushton and Krishnamurthi [66] and Rahman and Smith [57]. In both these studies, the problem of locating optimal sites was considered as a \( p \)-median problem. In their study, Rushton and Krishnamurthi [66] computed the locational efficiency of health facilities that had actually been chosen in the state of Karnataka, India during the period between 1971 and 1981. They found that the selected health facilities from 1971 to 1976 were 70% efficient compared to the optimal sites, 77% efficient in the period 1976–1979 and 62% efficient from 1979 to 1981.

The study by Rahman and Smith [57] was designed to find sites for Health and Family Welfare Centres (HFWC) in Tangail Thana in Bangladesh. HFWCs are meant to organise immunisation activities, treat diarrhoeal diseases and fever cases, and work for the family planning programmes in the rural areas. According to government policy, ten HFWCs are to be opened to serve the area. However, seven are already operating in the area. In addition, there are three rural dispensaries (RD) in the area which according to the government's plan will be upgraded and converted to HFWCs. This means that sites for 10 HFWCs have already been decided and there remained no scope for siting any new facilities. Therefore, the study concentrated on the following analysis:

- Compare the locational efficiency of seven existing HFWCs with seven optimal HFWCs.
- Compare the locational efficiency of seven existing plus three proposed HFWCs with seven existing plus three optimal HFWCs.

The optimal locations of seven facilities found by the \( p \)-median method meant that the mean distance traveled by users (the objective function in this case) was 1.9 km while the existing facilities needed an average journey of 3.1 km – 65% more. This study also demonstrated that in a system with four optimally located HFWCs, the number of person-kilometers travelled is approximately equal to the number of person-kilometers travelled in the existing system with seven facilities. The system with seven existing plus three proposed facilities produced a mean distance travelled of 2.3 km. In the system with seven existing plus seven optimal facilities found by the \( p \)-median method the mean fell to 1.7 km, an improvement of 26%. Other relevant studies include those by Logan [41] in Sierra Leone, Ayemi et al. [2] in Nigeria and Oppong and Hodgson [53] in Ghana.

The studies by Rushton and Krishnamurthi [66], Logan [41], Ayemi et al. [2] and Rahman and Smith [57] are examples of SLAMs. Some studies which belong to the HLAM category and which also addressed the efficiency issue are those by Fisher and Rushton [24] and Hodgson and Valadares [32]. Both these studies considered the location-allocation problem as a \( p \)-median problem and solved it using the Teitz and Bart [74] heuristic. While conducting location analysis for health facilities along with other services such as police stations and district headquarters in Junagadh, India, Fisher and Rushton [24] found that compared with the optimal system, the existing system was 1.22 times inefficient. This means that the average distance travelled in the existing system is 1.22 times that in the optimal system. In Hodgson and Valadares’ [32] study the spatial inefficiency of the existing health system in Goa, India was found to be even greater (1.56 times). In both these studies the problems were considered as a series of single-level, \( p \)-median problems, solved in a step manner (top-down or bottom-up approach). Hodgson [33] demonstrated that the simultaneous approach produces a better solution than either stepwise method.

4.4. Improving existing systems

Okafor [51] conducted a study to find a site (from four possible sites) for a hospital which is to be added to an existing health delivery system with
three hospitals in Bendel state, Nigeria. The problem was developed and solved as a transportation problem similar to one developed by Gould and Leinbach [27]. The capacities of the existing hospitals were included in the transportation formulation. However, it appears from the study that a $p$-median type formulation would have been more appropriate. With the limited choice, this was not an especially difficult problem, although there was scope for sensitivity analysis. Eaton et al. [20], on the other hand, used an MCLP method to locate facilities in Colombia. The purpose of the study was to find sites for new health centres which, if added to the 28 existing sites, would most improve population coverage. Mehretu [44] in a study in Burkina Faso used the $p$-median method to locate new health centres in an existing system that provided primary health care. However, the government planners imposed two constraints. These are: (1) every village with a population of 730 and over will be assigned one primary Community Clinic; (2) every rural household should have access to a health facility within a maximum of 5 km.

Mehrez et al. [46] conducted a study to locate a new hospital in Israel using location-allocation models in conjunction with the Analytic Hierarchy Process (AHP) approach. First the problem was analysed using the $p$-median method and LSCP both on the plane and on networks. Then the AHP was applied to evaluate the optimal sites using a set of criteria which include: minisum ($p$-median) objective function; service availability to remote settlements; improving employment; contribution for population diversity; using the existing infrastructure.

5. Conclusion and future research directions

The location-allocation models and methods reviewed in this paper have been formulated either as $p$-median problems or covering problems. The objective of the $p$-median problem is to locate a given number of facilities so that the total travel distance (or time) between facilities and demand points is minimised. Location analysis using $p$-median formulations seems to be one of the most popular approaches to rural health facility location planning in developing countries (the majority of the studies described have used this formulation). The objective of LSCP is to locate the minimum number of facilities such that each demand point has a facility within a given maximal service distance (or time) and the objective of MCLP is to locate a fixed number of facilities to maximise the total demand within a maximum service distance (or time). When LSCP was first introduced, its simplicity of problem statement and its straightforward solution technique quickly established it as a widely recognised model for the location of public facilities. Chaiken [11] reported that Toregas and ReVelle’s [77] model had been acquired by many national and international organisations.

An underlying assumption of most of the location models described in this paper is that the health facilities being sited are uncapacitated (i.e. each has an infinite capacity to serve consumer demand). This paper has identified three exceptions, one the paper by Okafor [51], that by Gould and Lienbach [27] where the location of hospitals was investigated but the capacity constraint made it insoluble, and the work of Heller et al. [30] on the capacitated $p$-median problem. Solving uncapacitated problems means that the work load at the facilities may vary substantially. It is typical of developing countries that rural health facilities are organised with similar medical equipment and employ the same number of health personnel. (For instance, each Community Clinic intended to deliver primary health care at the village/ward level in Bangladesh is to be manned by a Health Assistant and a Family Welfare Assistant [58].) Allowing variation in the demand for services at the facilities could be handled either by solving the problem directly as a capacitated location problem or by allocating health personnel according to the demand at the facilities. Recently, Current and Storbeck [13], and Pirkul and Schilling [55] have suggested capacitated versions of the classical covering models which would be useful for health facility location planning with capacity constraints in developing nations. For solving the capacitated $p$-median problem one can make use of location-allocation formulations suggested by Goodchild [26].
The studies reviewed in this paper have been applied to find optimal sites of facilities, assess the effectiveness of previous location decisions, and generate viable alternatives for action by decision makers. An assessment of the effectiveness of past locational decisions can provide information regarding what could be achieved using the same resources. This type of study has limited use, since to relocate an existing system even partly to improve efficiency could be infeasible both politically and economically in the context of developing nations. Referring to a study in Honduras, Moore ([48], p. 135), reported that relocation of auxiliary nurse posts would be politically infeasible since “the resentment created by closing facilities would probably be greater than any goodwill created by opening others”. In the absence of formal analysis for locational decisions, quite often final decisions are made by the local politicians. Evidence from the studies show that these decisions are generally far from optimal. In order to prevent such interventions locational models can play an important role in generating alternative location arrangements and predicting the cost-effectiveness of entirely new service systems or existing systems which are to be expanded using new facilities. The same view has been shared by Rushton ([68], p. 113): “So long as the ledger is blank, politicians have the opportunity to vie with one another to bring spoils back to their constituents, but with the accounts open and enough detail to count the costs and returns for alternative locations, politicians have to pause before they intervene to counter the (bureaucrats’) recommendations”.

Most of studies reviewed have used some kind of single criterion objective functions. In reality however, most location decisions are complex problems and require multi-criteria objective models. In a few of the cases described, secondary objectives have been included as constraints, as a simple way of handling multi-dimensional measures of performance. This may not always be adequate. It seems, therefore, that there is a necessity to develop multiple criteria location models to develop health systems in developing nations. This is in line with the findings of other researchers. For instance, Erkut and Neuman [23] mentioned that ‘current models can be used to generate a small number of candidate sites, but the final selection of a site is a complex problem and should be approached using multi-objective decision making tools’.

Understanding a problem and taking actions to improve the problem situation may follow the following steps:
1. Problem conceptualisation and definition.
2. Model (conceptual and quantitative) development.
3. Analysis of the model.
4. Evaluation of results.
5. Implementation of results.

Most of the studies reviewed in this paper have addressed Steps 1–4. Only a handful of studies claimed implementation. In spite of the great potential of the location-allocation models for developing and improving health systems, it is not clear why only a few studies were implemented. Rahman and Smith [59] suggested the following necessary conditions for proper implementation of OR studies in developing nations:
1. The involvement of local analysts from the beginning by attachment to the project team, irrespective of whether or not the project was initiated locally. The successful completion and implementation will depend on the knowledge and experience of such local people.
2. Effective communication with local decision-makers to demonstrate the benefits of using the OR approach.

Point 1 has been frequently advocated in the OR literature [60,69]. As regards to point 2 in the context of location-allocation studies in developing countries, some successful efforts were reported in [46,56,21,6]. However, both are to be considered as an integral part of any OR project in developing nations. Point 1 is necessary to allow for successful transfer of every phase of the planning and model for further use; point 2 is required for acceptance of the implementation and diffusion of OR ideas amongst planners and decision makers.

There will be health needs in the less-developed countries of the world for many years to come. The modelling skills of the operational research scientist in studying the location and use of health centres will continue to be of value to the planners and decision-makers. The same scientist’s communica-
tion skills will be essential to involve the consumers and their communities in the provision of suitably located facilities. This paper has shown that this has happened in the past, is happening now, and, hopefully, will continue to happen in the future.

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Appendix A

Based on the relationship between various levels of hierarchies, Tien et al. [75] recommended three types of service hierarchies. These are: successively inclusive, locally inclusive and successively exclusive hierarchies. A hierarchical system in which the facilities at any level offer all the services offered by the facilities of a lower level is said to have a successively inclusive hierarchy. This has been illustrated in Fig. 2. Here, the facility at location 4 (level 3 facility) offers type 1, 2 and 3 services to all locations and the facility at location 3 (level 2 facility) offers type 1 and 2 services to all locations. In health care systems it is generally assumed that the facility hierarchy is successively inclusive. However, there are examples which could be locally inclusive as well. In a locally inclusive hierarchy a facility at any level offers all services only to the location where it is located and only the highest order service to all other locations (Fig. 2 (ii)). Fig. 2 (ii) shows that the facility at location 4 (level 3 facility) offers type 1, 2 and 3 services to location 4 and offers only type 3 service to all other locations. Likewise, the facility at 3 (level 2 facility) offers type 1 and 2 services to location 3 and offers only type 2 services to other locations. The health care system discussed by Calvo and Marks [10] is an example of such an hierarchy. In a successively exclusive hierarchy a facility at level $m$ offers only type $m$ services to all locations. This has been illustrated in Fig. 2 (iii).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(i) Successively inclusive hierarchy. (ii) Locally inclusive hierarchy. (iii) Successively exclusive hierarchy.}
\end{figure}
Production–distribution systems and solid waste disposal systems are generally assumed to have successively exclusive hierarchies. In a production–distribution system a product is manufactured at a factory and stored at warehouses from which it is transferred to the retail shops. Here each facility offers only one type of service.

Narula [49] proposed a classification scheme based upon the number of types of facilities in a hierarchy and the flow of service between locations and types of facilities. The flow and the network can be divided into four categories. The flow may be integrated (I) or discriminating (D). A flow is said to be integrated if it occurs from any lower level 1, 2, …, f facility to any higher level 1, 2, …, f facility. A flow is discriminating when it occurs from any lower level facility m to the next higher level facility m + 1 only.

The network may be unipath (U) or multipath (M). In a unipath network the positive degree of every node is less than or equal to 1. In a multipath network the positive degree of at least one of the nodes is greater than or equal to 2. The degree of a node is the number of lines incident at that node.

These categories may be combined to give four options which are preceded by the number of facilities to yield

\( f/I/U \) location-allocation models.
\( f/I/M \) location-allocation models.
\( f/D/U \) location-allocation models.
\( f/D/M \) location-allocation models.

Appendix B

1. The \( p \)-median problem [62]

minimise \( \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij} \)

subject to \( \sum_{j=1}^{n} x_{ij} = 1 \quad \forall i, \)
\( \sum_{i=1}^{n} x_{ij} = p, \quad \forall j, \)
\( x_{ij} \leq x_{ji} \quad \forall i, j, \)
\( x_{ij} = 0, 1 \quad \forall i, j, \)

where \( x_{ij} = 1 \) if demand \( i \) is assigned to a facility \( j, \)
\( x_{ij} = 0 \) otherwise, \( n \) the number of demand points,
\( a_i \) the population of demand \( i, \)
\( d_{ij} \) the shortest distance between \( i \) and \( j, \)
\( p \) the number of facilities to be located.

As an integer-programming problem, there are \( n^2 \) zero–one variables, although reducing the number of possible sites for facilities can make this smaller.

2. The LCSP [78]

minimise \( \sum_{j=1}^{n} x_{j} \)

subject to \( \sum_{j \in \mathcal{N}_i} x_{j} \geq 1, \quad \forall i, \)
\( x_{j} = 0, 1 \quad \forall j, \)

where \( x_{j} = 1 \) if a facility is located at \( j, \)
\( x_{j} = 0 \) otherwise, \( \mathcal{N}_i = \{ j | d_{ij} \leq S \} \) is the set of facilities which are eligible to provide cover to demand \( i, \)
\( S \) the maximal service distance, \( n, d_{ij} \) are as before.

This problem only has \( n \) zero–one variables, which makes it suitable for large problems, although the \( n \) inequality constraints may make it computationally difficult.

3. The MCLP [12]

maximise \( \sum_{i=1}^{n} a_i y_i \)

subject to \( \sum_{j \in \mathcal{N}_i} x_{j} \geq y_i, \quad 1 \leq i \leq n, \)
\( \sum_{j=1}^{n} x_{j} = p, \)
\( x_{j} = 0, 1 \quad \forall j, \)
\( y_i = 0, 1 \quad \forall i, \)

where \( x_{j} = 1 \) if a facility is located at \( j, \)
\( x_{j} = 0 \) otherwise, \( y_i = 1 \) if demand from \( i \) is covered by a facility, \( y_i = 0 \) otherwise, \( \mathcal{N}_i = \{ j | d_{ij} \leq S \} \) is the set of facilities which are eligible to provide cover to demand \( i, \)
\( S \) the maximal service distance, \( n, a_i, d_{ij}, p \) are as before.

This problem only has \( 2n \) zero–one variables, which makes it suitable for large problems.
than the $p$-median problem. The constraints are “awkward”.

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