A Novel Mathematical Model for Robust Green Urban Waste Collection

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Outline

- Introduction
- Literature review
- Problem description/Mathematical model
- Robust optimization approach
- Augmented $\varepsilon$-constraint method
- Illustrative example
- Model validation
- Sensitivity analysis
- References
Introduction

• Uncontrolled urban expansion and the massive increase in urban populations have led to a vast amount of consumption and different types of waste generation over the past years.
Introduction

- In 2012, various cities in the world generated **1.3 billion** tons of solid waste, equivalent to **1.2 kg per person per day**.

https://www.statista.com
• It is expected that annual waste generation rate will reach 2.2 billion tons by 2025, accordingly.

https://www.statista.com
Introduction

- **Economical aspect:**

  - This amount of waste generation definitely leads to an increase in the necessary funds for *collection*, *transportation* and *disposal* operations, which contains 75%-80% of Urban Solid Waste Management (USWM) costs (Tirkolaee et al., 2018).
Introduction

- Environmental and health aspect:

  - These operational processes must be done within the *shortest possible time* to prevent the spread of potential contamination and infections.
  
  - On the other hand, there is now a global force on **GHG reduction**, which should be considered in USWM too.
Introduction

- **Uncertainty:**
  - The exact amount of generated waste is always unknown in each urban area due to its uncertain nature.
  - In fact, each area may have a lower and an upper bound for the amount of waste according to its minimum and maximum values in a given time period.
Questions:

1. How can we investigate the economical and environmental aspects of the problem?
   - Identifying the main parameters, variables and assumptions.
   - Formulate the problem using OR concepts and techniques.

2. How can we optimize the amount of required budget and the amount of contamination?
   - Developing a bi-objective mathematical model considering all the defined assumptions.
   - Implementing an efficient solution technique to deal with the bi-objectiveness of the problem; i.e., Augmented $\varepsilon$-constraint method.

3. How can we evaluate the effects of uncertainty?
   - Applying Robust Optimization (RO) approach.
   - Performing sensitivity analyses.

Introduction
A robust bi-objective model is proposed for the Multi-Trip Vehicle Routing Problem with Time Windows (MTVRPTW) to concurrently minimize the total cost and the total GHG emission.
## Literature Review

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Consider an urban graph network which represents the demand nodes, parking and disposal sites:

Black circles: Household waste,
Green circles: Hazardous Waste like hospital waste.
Assumptions

- The main assumptions of the problem are as follows:
  - Each demand node is **only** serviced by one vehicle.
  - There is **one** depot and **one** disposal site.
  - Vehicles are **heterogeneous** and have a **maximum allowable time of service**.
  - Vehicles may have **multiple trips**.
  - **Traversing cost of a route** is the same for all vehicles.
  - Each demand node has a **hard** and **soft time window of service**, which means that violating a **soft time window** will have a penalty, and violating a **hard time window** is not possible.
Multiple trips in USWM

- Consider 8 nodes including 1 depot, 1 disposal site and 6 demand nodes:

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<th>1-2-4-7-8</th>
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<td>Second trip</td>
<td>8-6-5-3-8</td>
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The main objectives of the problem are represented as follows:

- **Minimizing** total traversing cost, total usage cost of vehicles and total penalty cost of time windows violation
- **Minimizing** total GHG emissions of vehicles

**Economically and Environmentally Sustainable Waste Collection System**
### Mathematical Model

minimize \( Z = \theta \left( \sum_{i \in NT} \sum_{j \in NT} \sum_{k \in K} \sum_{r \in R} c_{ij} x_{ijk}^r \right) + \sum_{k \in K} \sum_{i \in NC} (P e_k + P l Y_{ki}^l) + \sum_{k \in K} c v_k u_k \)

- **1\textsuperscript{st}** objective function is to minimize the total cost including *traversing costs, the penalty costs for violating permissible time windows* and *the usage costs of vehicles*.

\[
\text{minimize } G = \sum_{i \in NT} \sum_{j \in NT} \sum_{k \in K} \sum_{r \in R} g_h g_{ijk} x_{ijk}^r
\]

- **2\textsuperscript{nd}** objective function indicating the total amount of emissions.

subject to

\[
\sum_{j \in NT} x_{ijk}^r = \sum_{j \in NT} x_{jik}^r \quad \forall i \in NT, \forall k \in K, \forall r \in R,
\]

- The flow balance of each vehicle.
Mathematical Model

\[ \sum_{k \in K} \sum_{r \in R} y_{jk}^r = 1 \quad \forall j \in NC, \]

- Each demand node is only serviced by one vehicle:

\[ \sum_{j \in NC} d_j y_{jk}^r \leq W_k \quad \forall k \in K, \forall r \in R, \]

- Each vehicle has a capacity constraint.

\[ y_{jk}^r \leq x_{ij}^r \quad \forall i, j \in NT, \forall k \in K, \forall r \in R, \]

- The demand node is served by the vehicle which has already been arrived at there.

\[ \sum_{(i,j) \in NE} \sum_{r \in R} x_{ijk}^r \leq M u_k \quad \forall k \in K, \]

- Vehicles can be used when their costs are paid.
Mathematical Model

\[ LT_k^r = ul \sum_{j \in NC} d_j y_{jk}^r \quad \forall k \in K, \forall r \in R, \]

\[ UT_k^r = uu \sum_{j \in NC} d_j y_{jk}^r \quad \forall k \in K, \forall r \in R, \]

- The total loading and unloading time for each vehicle per trip.

\[ \sum_{r \in R} LT_k^r + \sum_{r \in R} UT_k^r + \sum_{(i,j) \in NE} \sum_{r \in R} t_{ij} x_{ijk}^r \leq T_{max} \quad \forall k \in K, \]

- The usage time limitation of each vehicle.

\[ \sum_{i \in S} \sum_{j \in S} \sum_{k \in K} x_{ijk}^r \leq |S| - 1 \quad \forall S \subseteq NC; |S| \geq 2, \forall r \in R, \]

- Sub-tour elimination constraint of each vehicle.
Mathematical Model

\[
\sum_{j \in NC} x_{1jk}^1 \geq \sum_{j \in NC} x_{njk}^2 \quad \forall k \in K,
\]
\[
\sum_{j \in NC} x_{njk}^r \geq \sum_{j \in NC} x_{njk}^{r+1} \quad \forall r \in \{2, 3, ..., |R| - 1\}, \forall k \in K,
\]

- The order number of vehicle trips is from \( r \) to \( r+1 \) in succession.

\[
\begin{align*}
    tt_j &= \sum_{i \in NT} \sum_{k \in K} \sum_{r \in R} (tt_i + t_{ij}) x_{ijk}^r \quad \forall j \in NC, \\
    tt_i &= 0 \quad \forall i = 1, \\
    e_i &\leq tt_i \leq l_i \quad \forall i \in NC,
\end{align*}
\]

- The arrival time of vehicles at each demand node and the **hard time window** is defined for each demand node.
Mathematical Model

\[ Y_{e_{ki}} \geq e_{e_i} - t_{t_i} \quad \forall k \in K, \forall i \in NC, \]

\[ Y_{l_{ki}} \geq t_{t_i} - l_{l_i} \quad \forall k \in K, \forall i \in NC, \]

- **Soft time window**: Calculation of earliness and lateness in serving each demand node.

\[
\sum_{j \in NC} x^1_{1jk} = u_k \quad \forall k \in K,
\]

\[
\sum_{i \in NC} x^1_{1ni_k} = u_k \quad \forall k \in K,
\]

- Vehicles’ first trip starts from the depot and ends at the disposal site.
Mathematical Model

\[ \sum_{i \in NC} x_{i,nk}^r \leq M(1 - u_k) + 1 \quad \forall r \geq 2, \forall k \in K, \]
\[ \sum_{j \in NC} x_{n,jk}^r \leq M(1 - u_k) + 1 \quad \forall r \geq 2, \forall k \in K, \]

- Other trips of vehicles (except for first trip) will begin from the disposal site and end at the disposal site.

\[ x_{n,1k}^r \geq x_{n,1k}^{r-1} \quad \forall r \geq 2, \forall k \in K, \]
\[ \sum_{r \in R} x_{n,1k}^r = u_k \quad \forall k \in K, \]

- Vehicles’ last trip end at the depot again to complete their tours:

\[ x_{i,jk}^r, y_{jk}^r, u_k \in \{0,1\}, \quad Ye_{ki}, Yl_{ki}, tt_i, LT_k^r, UT_k^r \geq 0 \quad \forall (i,j) \in NE, \forall k \in K, \forall r \in R. \]

- Types of variables.
Robust Optimization

- They claimed that it is very rare that at all the uncertain parameters of a constraint take different values far from their nominal and at their limit values simultaneously:

  ✓ For a parameter of \( z_{ij} = \frac{a_{ij} - \bar{a}_{ij}}{\hat{a}_{ij}} \in [-1, +1]; \sum_j |z_{ij}| \leq \Gamma_i \quad \forall i, \Gamma_i \in \left[ 0, |J_i| \right]. \)

  ✓ \( J_i \) is the set of uncertain parameters in the \( i \)-th row of the coefficient matrix of constraints.

  ✓ The constraints’ coefficients are uncertain, taking values in an interval with the center of \( \bar{a}_{ij} \) and the radius of \( \hat{a}_{ij} \); i.e., \( a_{ij} \in [\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}] \).
Robust Optimization

- $\Gamma_i$ represents the conservatism level and is known as the budget on uncertainty.

- $\Gamma_i$ is determined by a decision-maker and its value is dependent on the risk-aversion and the importance of the constraint for the decision-maker:

$$\Gamma_i = 0,$$

The robust model turns into a deterministic model

$$\Gamma_i = |J_i|$$

The robust model will be equal to the model proposed by Soyster (1973); uncertain coefficients take their worst possible values
Bertsimas and Sim approach

- The proposed optimization model by Bertsimas and Sim (2004):

\[
\begin{align*}
\text{minimize} \quad & Z = \sum_j c_j x_j \\
\text{subject to} \quad & \sum_j a_{ij} x_j \leq b_i, \quad \forall i, \\
& l_j \leq x_j \leq u_j, \quad \forall j.
\end{align*}
\]

\[
\begin{align*}
\text{minimize} \quad & Z = \sum_j c_j x_j \\
\text{subject to} \quad & \sum_j \tilde{a}_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} r_{ij} \leq b_i, \quad \forall i, \\
& z_i + p_{ij} \geq \tilde{a}_{ij} E_j, \quad \forall i, j, \\
& -E_j \leq x_j \leq E_j, \quad \forall j, \\
& l_j \leq x_j \leq u_j, \quad \forall j, \\
& p_{ij}, z_i, E_j \geq 0, \quad \forall i, j.
\end{align*}
\]

- Here, \( r_{ij}, z_i \) and \( E_j \) are the \textit{auxiliary dual variables} that are used to prevent from non-linearization of the problem.
Robust Optimization Model

- Since the amount of waste; i.e., demand parameter \((d_j)\) leads to the uncertainty, the capacity constraint should be developed based on Bertsimas’ and Sim’s formulation:

\[
\sum_{j \in NC} d_j y_{jk}^r \leq W_k \quad \forall k \in K, \forall r \in R,
\]

\[
\sum_{j \in NC} \tilde{d}_j y_{jk}^r + z_{kr} \Gamma_{kr} + \sum_{(i,j) \in J_{kr}} r r_{jk}^r \leq W_k \quad \forall k \in K, \forall r \in R,
\]

\[
\begin{align*}
z_{kr} + r r_{jk}^r & \geq \hat{d}_j E_{jk}^r & \forall k \in K, \forall r \in R, \forall j \in NC, \\
E_{jk}^r & \leq y_{jk}^r & \leq \bar{E}_{jk}^r & \forall k \in K, \forall r \in R, \forall j \in NC, \\
E_{jk}^r, E_{jk}^r, z_{kr} & \geq 0 & \forall k \in K, \forall r \in R, \forall j \in NC.
\end{align*}
\]

\[
d_j \in [\tilde{d}_j - \hat{d}_j, \tilde{d}_j + \hat{d}_j], \quad \hat{d}_j = \rho \cdot \tilde{d}_j \quad \forall j \in NC; \quad \rho \text{ is uncertainty level.}
\]

- Note that the other parts of the proposed model will remain unchanged.
Augmented $\varepsilon$-constraint method

- **Augmented $\varepsilon$-constraint technique** was proposed by Mavrotas (2009) to generate efficient Pareto solutions in multi-objective mathematical programming:

  - Lexicographic approach is used to determine the range of objective functions’ values (as its benefit compared to the traditional $\varepsilon$-constraint technique).

  - Then, the augmented $\varepsilon$-constraint only generates **efficient** Pareto fronts and prevents from inefficient ones.
Formulation of the augmented ε-constraint:

\[
\text{maximize/ minimize} \left( f_1(\overline{x}) + \varepsilon d_i \sum_{i=2}^{p} \left( s_{ik} / r_{ai} \right) \right)
\]

subject to

\[
f_i(\overline{x}) - d_i s_{ik} = e_{ik} \quad s_{ik} \in R^+, \quad i = 2, 3, ..., p, \quad k = 0, 1, ..., q_i,
\]

\[
e_{ik} = f_i^{\min} (d_i + 1) / 2 - f_i^{\max} (d_i - 1) / 2 + d_i r_{ik} / q_i \quad i = 2, 3, ..., p, \quad k = 0, 1, ..., q_i.
\]

\[\begin{array}{|l|}
\hline
p & \text{The sub-objective function} \\
\hline
d_{i1} & \text{Direction of } i\text{-th objective (i.e., -1 for minimization type and +1 for maximization type)} \\
\hline
e_{ik} & \text{Changing parameter to obtain efficient solutions} \\
\hline
s_{ik} & \text{Slack variable} \\
\hline
\varepsilon & \text{Very small value between 0.001 and 0.000001} \\
\hline
\end{array}\]

Mavrotas and Florios (2013)
Augmented $\varepsilon$-constraint method

$\Phi = \begin{bmatrix} f_1^*(\bar{x}_1) & \cdots & f_p^*(\bar{x}_1) \\ \vdots & \ddots & \vdots \\ f_1^*(\bar{x}_p) & \cdots & f_p^*(\bar{x}_p) \end{bmatrix}$

Start

Input model parameters

Pay-off table calculation using Lexicographic Optimization

Determining the range of objective functions

Dividing the range of objective functions ($i=2,\ldots,p$)

$\begin{array}{l}
i = 2 \\
n_i = 0 \\
n_i = n_i + 1 \\
i = i + 1 \\
\text{Form single objective optimization subproblem on the basis of } \varepsilon_n \\
\text{Solve } i-n_i \text{ subproblem which generates a Pareto optimal solution}
\end{array}$

Yes

No

$nl < q_i$

$nl < q_i$

$i < p$

Pareto optimal set

$\min F(\bar{x}) = (f_1(\bar{x}), \ldots, f_p(\bar{x}))^T$

s.t. $\bar{x} \in \Omega$

$f_i^{SU} \leq f_i(\bar{x}) \leq f_i^{SN}$

$f_i^{SN} = \max \left\{ f_i(\bar{x}_1^r), f_i(\bar{x}_2^r), \ldots, f_i(\bar{x}_p^r) \right\}$

$f_i^{SU} = f_i^*(\bar{x}_i^r)$

Aghaei et al. (2011)
Consider a town with an area of 330 square kilometers including a vehicle parking as the depot, a disposal site and 43 demand nodes,

The values of the parameters are determined randomly using uniform distribution,

The uncertainty level ($\rho$) and conservatism level ($\Gamma_{kr}$) are set to be 0.2 and 5, respectively.
Model Validation

- To validate the proposed robust bi-objective MILP model, CPLEX solver of GAMS 24.1 is employed. The execution steps of the augmented $\epsilon$-constraint method to provide efficient Pareto solutions are as follows:

**Lexicographic payoff table**

<table>
<thead>
<tr>
<th>Single-objective minimization</th>
<th>1$^{\text{st}}$ objective value</th>
<th>2$^{\text{nd}}$ objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^{\text{st}}$ objective</td>
<td>98346.690</td>
<td>117.824</td>
</tr>
<tr>
<td>2$^{\text{nd}}$ objective</td>
<td>117532.664</td>
<td>79.203</td>
</tr>
</tbody>
</table>

- In the next step, the subsidiary objective function (2$^{\text{nd}}$ objective function) is divided into 8 equal intervals (grid points):

<table>
<thead>
<tr>
<th>Grid points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>79.203</td>
</tr>
</tbody>
</table>
Model Validation

- We can generate all the efficient Pareto solutions using different grid points:

<table>
<thead>
<tr>
<th>Pareto solution</th>
<th>Obj 1</th>
<th>Obj 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>112532.664</td>
<td>79.203</td>
</tr>
<tr>
<td>2</td>
<td>108251.113</td>
<td>83.720</td>
</tr>
<tr>
<td>3</td>
<td>102659.600</td>
<td>89.664</td>
</tr>
<tr>
<td>4</td>
<td>101006.099</td>
<td>95.775</td>
</tr>
<tr>
<td>5</td>
<td>100114.163</td>
<td>100.584</td>
</tr>
<tr>
<td>6</td>
<td>100030.486</td>
<td>103.257</td>
</tr>
<tr>
<td>7</td>
<td>100015.359</td>
<td>108.668</td>
</tr>
<tr>
<td>8</td>
<td>99816.297</td>
<td>114.260</td>
</tr>
</tbody>
</table>

- Now, decision-maker can choose his/her most preferred solution among these 8 efficient Pareto solutions. It is represented by a green arrow.
Sensitivity analysis

- Here, we analyze different conditions in the problem to simulate and study the instability of the real-world. To this end, sensitivity analyses are done on the maximum available time of vehicles ($T_{max}$) and uncertainty level ($\rho$).
- Note that the results are obtained by the grid point corresponding to the best Pareto solution.

<table>
<thead>
<tr>
<th>Objective functions</th>
<th>Change interval of $T_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>First objective</td>
<td>113828.095</td>
</tr>
<tr>
<td>Second objective</td>
<td>92.058</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Objective functions</th>
<th>Change interval of $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>First objective</td>
<td>101298.382</td>
</tr>
<tr>
<td>Second objective</td>
<td>88.638</td>
</tr>
</tbody>
</table>
Sensitivity analysis

• The above figures can be investigated in order to be employed as effective managerial tools.

• As it is obvious, the behaviors of the objectives are corresponding to each other against the changes of the parameters.
Conclusion

- Optimal routing and vehicle allocating are two of the most important decisions of organizations such as municipalities for urban waste collection.

- Establishing an optimal waste collection system leads to an economically and environmentally sustainable waste management.

- Uncertainty in the waste amount plays an important role to estimate the required resources for waste collection.

- Robust optimization approach could provide feasible solutions under different uncertain conditions.
Augmented $\varepsilon$-constraint technique could efficiently generate Pareto solutions and represent the trade-off between the economical and environmental aspects of the problem.

Objective functions have a significant sensitivity to the maximum available time of vehicles and uncertainty level.

The 1$^{\text{st}}$ and 2$^{\text{nd}}$ objectives increase by an increase of the uncertainty level, and decrease by a decrease of the maximum available time of vehicles.
Future Research Directions

- For future studies, the following items are suggested based on the main limitations of the research:
  
  - **Heuristic and metaheuristic** methods may be implemented to solve the problem in larger sizes efficiently such as those proposed by Babaee Tirkolaee et al. (2019), Goli et al. (2019).
  
  - Other uncertainty techniques such as **fuzzy theory** (Çalık et al., 2018), **grey systems** (Roy et al., 2017) and **stochastic optimal control** (Temoçin and Weber, 2014) may be compared to the current robust optimization approach.
  
  - Other objective functions including distribution reliability maximization or customers’ satisfaction maximization can be studied, too.
## Notations:

<table>
<thead>
<tr>
<th>Sets and indices</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NC$</td>
<td>Set of nodes with demand (demand node)</td>
</tr>
<tr>
<td>$NT$</td>
<td>Set of total nodes</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of vehicles</td>
</tr>
<tr>
<td>$R$</td>
<td>Set of vehicle trips</td>
</tr>
<tr>
<td>$S$</td>
<td>Any optional subset of the demand node set</td>
</tr>
<tr>
<td>$i, j$</td>
<td>Demand node index</td>
</tr>
<tr>
<td>$k$</td>
<td>Vehicle index</td>
</tr>
<tr>
<td>$r$</td>
<td>Trip index</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{ij}$</td>
<td>Traversing cost of edge $(i,j)$</td>
</tr>
<tr>
<td>$Pe$</td>
<td>Early service cost per demand node</td>
</tr>
<tr>
<td>$Pl$</td>
<td>Late service cost per demand node</td>
</tr>
<tr>
<td>$(ei_{i},li_{i})$</td>
<td>Hard time window of demand node $i$</td>
</tr>
<tr>
<td>$(ee_{i},ll_{i})$</td>
<td>Soft time window of demand node $i$</td>
</tr>
<tr>
<td>$W_k$</td>
<td>Capacity of vehicle $k$</td>
</tr>
<tr>
<td>$d_{j}$</td>
<td>Demand of node $j$</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>Maximum available time for each vehicle</td>
</tr>
<tr>
<td>$M$</td>
<td>Optional large number</td>
</tr>
<tr>
<td>$ghg_{ijk}$</td>
<td>Amount of GHG emission by vehicle $k$ when traversing edge $(i,j)$</td>
</tr>
<tr>
<td>$ul$</td>
<td>Unit of loading time for vehicles at demand nodes</td>
</tr>
<tr>
<td>$uu$</td>
<td>Unit of discharging time for vehicles at the disposal site</td>
</tr>
<tr>
<td>$t_{ij}$</td>
<td>Time for traversing edge $(i,j)$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Converter coefficient of distance to cost ($/km$)</td>
</tr>
</tbody>
</table>
Mathematical Model

Notations:

<table>
<thead>
<tr>
<th>Non-decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$tt_i$</td>
</tr>
<tr>
<td>Arrival time at demand node $i$</td>
</tr>
<tr>
<td>$LT_k^r$</td>
</tr>
<tr>
<td>Total loading time of vehicle $k$ in trip $r$</td>
</tr>
<tr>
<td>$UT_k^r$</td>
</tr>
<tr>
<td>Total unloading time of vehicle $k$ in trip $r$</td>
</tr>
<tr>
<td>$Ye_{ki}$</td>
</tr>
<tr>
<td>Amount of early service of the demand node $i$ by vehicle $k$</td>
</tr>
<tr>
<td>$Yl_{ki}$</td>
</tr>
<tr>
<td>Amount of late service of the demand node $i$ by vehicle $k$</td>
</tr>
</tbody>
</table>

Decision variables

$$x_{ijk}^r = \begin{cases} 
1 & \text{If vehicle $k$ moves from node $i$ to node $j$ in trip $r$ without serving node $j$,} \\
0 & \text{Otherwise.}
\end{cases}$$

$$y_{jk}^r = \begin{cases} 
1 & \text{If vehicle $k$ serves node $j$ in trip $r$,} \\
0 & \text{Otherwise.}
\end{cases}$$

$$u_k = \begin{cases} 
1 & \text{If vehicle $k$ is used,} \\
0 & \text{Otherwise.}
\end{cases}$$
References

References

Thank you very much for your attention!

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e.babaee@ustmb.ac.ir