Eigen-behaviors in Closed Economies

Keywords: Price formation, eigen-behaviors, principle of closure, general equilibrium, business attractor

By

Gabriel J. Turbay, Ph.D.*

* Presidente de la Sociedad Científica Pensamiento Sistémico y Teoría de Juegos de Estrategia, Sociedad Colombiana de Economistas, Carrera 20 No 36-41. Bogotá D.C., Colombia, Tel. (57) 1 695 5842. Cel.(57) 315 803 3701. e-Mail-1: economistascolombia@cable.net.co, e-Mail-2: gt.gabrielturbay@gmail.com.

Abstract

Costs and prices are essential components in added value processes and their dynamic interaction is highly relevant in determining industry profitability. In this paper, social phenomena are viewed as a closed system whose behavior converges to certain stability. Based on Heinz von Foerster principle of closure that states that in every operationally closed system, eigenbehaviors occur, by using Krilov sequences and standard matrix theory results, the dynamic recursive feedback process: costs-prices, prices-costs, that occurs in the natural formation of prices through time in an economy with n-industries and n-products is shown to converge to the respective eigenvectors with special equilibrium and resilience properties. Thus, an important validation of the closure principle is made to exhibit and understand a fundamental business and economic attractor and in doing so, a constructive proof of general economic equilibrium is obtained.

Eigen-Behaviors in Closed Economies

"*In order to know anything it is necessary to know everything, but in order to talk about* anything it is necessary to neglect a great deal."
Joan Robinson

1. Introduction.

During the 1990´s, in several of his magical lectures around the world, Heinz von Foerster[1] stressed the point that **s**ocial phenomena should be looked upon as a closed system whose behavior converges to certain stability. The principle of closure, he pointed out, states that in every operationally closed system, eigen-behaviors occur. From this perspective, human institutions, creations and developments like the nation-state, the army, the church, the language, learning processes, corruption, the world concentration of wealth and even products such as a piano or a car, emerge as the result of iterations and feedback processes in closed systems.

Before World War II, Wassily Leontief´s model of a stati*c* economic structure was a closed-end system. For the matrix of this closed system he proved in 1952 the existence of a maximum eigen-value. During and after the War, the model was then opened, and consumption was treated as a variable, not as the input of labor, mainly because the opened-end model was more amenable for prediction purposes[2]. In a closed model every output becomes an input and when a feedback process of inputs into outputs and outputs become inputs takes place, the principle of closure operates and one can expect eigen-behaviors to emerge. In the dynamic interpretation of Leontief˝s model, in Sraffa's model of production of commodities by means of commodities and in von Neumann´s pioneering model of a linear expanding economy, equilibrium convergence occurs under certain

[1] Heinz von Foerster worked with Norbert Wiener, Warren McCulloch, and Jonh von Neumann in the development of cybernetics and was the developer of **second-order cybernetics** which focus on self-referential systems and the importance of eigen-behaviors for the explanation of complex phenomena.
[2] A detailed treatment of this point can be found in Dorfman, Samuelson and Solow (1958)

conditions, and all these models exhibit what here will be identified as eigen-behaviors. In all cases the Ricardian invariant standard of value appears to be related to the left eigen-vector of the production matrices of these models.

The concept of prices in duality theory were identified as “left vectors” of the production or technology matrices and a particular left eigen-vector of these matrices was identified as the Invariable measure of value by Goodwin (1983) and others. The term eigen-prices was formally introduced by Seton (1981) and related work has been developed by Akura (1993) and Dietzenbacher and Wagener(1999) among several.

In this paper, we demonstrate that a recursive process which occurs in a natural manner in the economic formation of prices and in the requisition of resources in a closed economy, converge to the eigen-vector of prices, and the eigen-vector of inputs with special general equilibrium, optimality and regulatory properties for the economic growth and development of the system. I n doing so, an important validation of Heinz von Foerster´s remarkable observations is accomplished. With a two significant features not previously incorporated in mathematical economic models: First, the general equilibrium obtained, results as an emergent property of the a natural feedback process in the closed system where, in every new transformation cycle, prices enter as costs and outputs as inputs. In previous models of economic growth, general equilibrium is mainly an non-constructive existence property, shown with the help of topological fixed point theorems. Second, the model is amenable for game theoretical considerations in perturbations and resilience of price behavior analysis.

Viewed as a system, the world economy is a closed one. The outputs of the system in a general sense become the inputs of a transformation continuum that converts resources into products and economic results. Also prices are fed back as costs in the economy. All the conditions for the principle of closure to operate and for eigen-behaviors to emerge are given. This suggests input-output analysis and closed systems approach as ideal tools for studying processes in the actual context of financial globalization of the world economy.

It may be that the economic importance of these results, especially in the field of value theory, rest in the fact that a general law of price determination is identified from a macro structural point of view. Prices are affected by costs and costs by prices and that permanent dynamic feedback affected by the interrelation of activities and the production structure defined by the technology, according to the principle of closure, leads to the fact that prices in the long run must have a stable behavior or eigen-behavior and its fluctuations are precisely the exception not the rule; an exception generated by many micro factors like innovation, greed and other perturbations and natural events beyond economics general intentions like the optimal allocation of resources. These exceptions are regulated by laws that also can be identified and exhibited at a micro level with the use of related tools like the ones provided by the theory of games of strategy developed to explain economic “rational” behavior in situations of conflict and cooperation. Nonetheless, despite disturbances at the micro level which may be the subject of a different discourse; it is shown here that the macro structure of a closed economy determines the intrinsic formation of prices as a component of a special type of general equilibrium.

The demonstration that a “natural” price formation process in a closed economy converges to a stable equilibrium begins with the observation that Krilov sequences for productive economies correspond to a natural way of accounting for the formation of the price of a product obtained as the cost of production incurred plus a profit. The demonstrations are self explanatory

2. Activities, technology matrices, closed economies and eigen-behaviors

One of the major advances in systems thinking and mathematical economics in the twentieth century was the development of Linear Programming and Linear Economic Models. These analytical tools were built upon basic units or elementary functions which T.C. Koopmans called activities. The concept of activity is the equivalent to the concept of process first introduced by John von Neumann (1937), in his model of a linear expanding economy. A graphic representation of an activity is the black box where the entering arrows represent the amounts of inputs required to produce one unit of output, which is in turn, represented by a box exiting arrow, the corresponding quantitative description is given by a vector of requirements to produce a unit of a product. The aggregate of such vectors of production processes that may share common and limited resources conform the technology matrix characteristic in linear optimization models. This technology matrix provides the basic structure of the closed systems studied here.

In general eigen-vectors and eigen-values emerge as the solutions to a wide variety of problems in mathematics, physics, engineering and economics. The term “eigen-behavior” appears to have its origin as a derivation of the eigen-values and eigen-vectors in matrix theory. The concept as a phenomenon has been identified to occur in complex biological processes, in chaos theory like in Mandelbrot attractors, and also related to systemic self-determination and self-organization. Varela (1979) gave de name eigen-behavior to the process of systemic self-determination.

The mathematical results used in the demonstrations are standard in matrix algebra and in mathematical economics. These results include the Perron-Forbenius theorems, eigenvector convergence of Krilov sequences, and other basic facts and assumptions.

3. Basic assumptions

Let A be an n x n technology matrix with coefficients $a_{ij} \geq 0$, where a_{ij} denotes the amount of good i required to produce one unit of good j , $N = \{1,2,...,n\}$, $i \in N$, $j \in N$. Closed production economies are characterized by the fact that every good produced is produced as an output but used as an input to generate at least one of the outputs according to the requirements established by the technology matrix A. The output of the economy is assumed to be produced in cycles or uniform periods of time.

Let x be a nonnegative n-vector $X \geq 0$, where X_j is the jth component of X and denotes the amount produced of good j.

Let $P \geq 0$ be a price n-vector where P_j denotes the price of the jth good, and let $B \geq 0$ be the initial available existence of goods in the economy.

Any vector X is feasible if and only if

$$AX \leq 0$$

Here AX is the input vector required to produce the output X.

We are interested mainly in technology matrices A and output vectors X for which

$$X \geq AX \geq 0$$

4. Forbenius theorems and Krilov sequences

Technology matrices are non-negative and have been studied thoroughly in relation to Input / Output Analysis related to the Leontief Model.

Since 1912 Forbenius results for an n x n non-negative matrix Q established the following

1. Q has an eigen-value $\lambda \geq 0$
2. An eigenvector $X \geq 0$ can be associated with λ
3. If $QX \geq qX$ for any number q and $X \geq 0$ then $\lambda \geq q$
4. If r is any eigen-value of Q then $\lambda \geq |r|$
5. If $Q_1 \geq Q_2$ then $\lambda_1 \geq \lambda_2$

The eigen-value λ of maximum modulus is known as the Forbenius root of Q

A Krilov sequence for a matrix Q is a recursion obtained from an initial arbitrary non-zero column vector $Z_{(0)}$ so that

$$Z_{(1)} = QZ_{(0)} = Q^1 Z_{(0)}$$

$$Z_{(2)} = QZ_{(1)} = Q^2 Z_{(0)}$$

$$. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; .$$

$$Z_{(n+1)} = QZ_{(n)} = Q^n Z_{(0)}$$

In general, and specifically for a given Technology Matrix Q and an initial vector $Z_{(0)}$, the corresponding Krilov sequence converge[3] to the eigenvector corresponding to the largest eigen-value in absolute terms (Forbenius root), so that

$$Z_{(n+1)} = QZ_{(n)} \approx \lambda Z_{(n)}$$

This convergence works also for left multiplication of Q by a row vector $Y_{(0)}$.

5. The Recursive Dynamics of Price Formation

If $P_{(0)}$ is an initial n-vector of unitary prices for the n commodities in the economy, then the unitary cost vector

$$C_{(1)}^T = \left(c_{1(1)}, c_{2(1)}, \ldots, c_{(1)}\right)$$

that would apply to the first production cycle is given by

$$C_{(1)}^T = P_{(0)}^T A$$

The unitary prices in the next production cycle would be given by the cost in the preceding cycle plus a profit margin α, assumed for now to be the same for all commodities, so that

$$P_{(1)}^T = (1+\alpha)C_{(1)}^T, \alpha \geq 0$$

Here α is the percentage of increment over the unitary cost.

Thus, the dynamic recursion of price formation can be written as follows:

$$C_{(1)}^T = P_{(0)}^T A \Rightarrow P_{(1)}^T = (1+\alpha)C_{(1)}^T$$

$$C_{(2)}^T = P_{(1)}^T A \Rightarrow P_{(2)}^T = (1+\alpha)C_{(2)}^T$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$C_{(n)}^T = P_{(n-1)}^T A \Rightarrow P_{(n)}^T = (1+\alpha)C_{(n)}^T$$

$$C_{(n+1)}^T = P_{(n)}^T A \Rightarrow P_{(n+1)}^T = (1+\alpha)C_{(n+1)}^T$$

Expressing $P_{(n)}$ in terms of $P_{(0)}$ we obtain

$$P_{(n)}^T = (1+\alpha)^n P_{(0)}^T A^n$$

---

[3] The proof of this convergence can be readily seen when the matrix Q has n linearly independent eigenvectors.

But the term $(P_{(0)}^{T} A^{n})$ represents the nth term of the Krilov sequence

$$Y_{(1)}^{T} = P_{(0)}^{T} A$$

$$Y_{(2)}^{T} = Y_{(1)}^{T} A = P_{(0)}^{T} A^{2}$$

....

$$Y_{(n)}^{T} = Y_{(n-1)}^{T} A = P_{(0)}^{T} A^{n}$$

$$Y_{(n+1)}^{T} = Y_{(n)}^{T} A = P_{(0)}^{T} A^{n+1}$$

For n sufficiently large, $Y_{(n)}^{T}$ is the positive eigen-vector of the non negative matrix A corresponding to its positive Forbenius root, and hence

$$Y_{(n)}^{T} A = \lambda Y_{(n)}^{T}$$

So that

$$P_{(n)}^{T} A = \left[(1+\alpha)^{n} P_{(0)}^{T} A^{n}\right] A$$

$$= (1+\alpha)^{n} Y_{(n)}^{T} A$$

$$= (1+\alpha)^{n} \lambda Y_{(n)}^{T}$$

$$= \lambda (1+\alpha)^{n} Y_{(n)}^{T}$$

$$= \lambda P_{(n)}^{T}$$

Which shows that $P_{(n)}^{T}$ is also the positive eigen-vector corresponding to the eigen-value λ and that differs from $Y^{T}{}_{(n)}$ in scale only.

The fact that Krilov sequences converge implies that the natural formation of prices whenever a natural feedback of prices into cost occurs, converges to a stable pattern of prices in a closed economy. Here the only assumption that can be questioned is the fact that the profit rate is assumed to be the same for all activities

6. Price Stability

Given a technology matrix A of a given closed economy, suppose y^T is an eigen-price vector, so that $y^TA = \lambda y^T$. Suppose that at certain moment in time the economy has reached a state where the prices are eigen-prices, that is

$$P_{(n)}^T = Y^T ,$$

then the unitary cost vector for the subsequent period is given by

$$C_{(n+1)}^T = Y^T A = \lambda Y^T$$

So that

$$P_{(n+1)}^T = (1+\alpha)C_{(n+1)}^T$$

$$= (1+\alpha)\lambda Y^T$$

If the closed economy in consideration is productive, λ must satisfy the condition $0 < \lambda < 1$ or $\lambda = {}^{1}\!/_{(1+\beta)}, \beta > 0$, where β is the rate of growth of the economy.

Then

$$P_{(n+1)}^T = \left[{}^{(1+\alpha)}\!/_{(1+\beta)}\right] Y^T$$

and

$$P_{(n+1)}^T = Y^T = P_{n+1}^T, whenever \alpha = \beta$$

That is, if the economy has reached the "natural" state where the unitary prices are given by a left eigen-vector of the technology matrix A, if the markup margin or rate of profit α is set equal to the rate of growth β of the economy, then the prices remain invariant through time.

7. The Recursive Dynamics of Production Programs

So far we have not considered the output and resources side of the economy for the process of finding a "natural" production path is not as straight forward as is the price formation process, however if we follow the input output sequence of the closed economy through time:

$$
\begin{array}{ll}
\text{OUTPUT} & \text{INPUT} \\
X_{(n+k)} & AX_{(n+k)} = X_{(n+k-1)} \\
\ldots\ldots & \ldots\ldots\ldots\ldots \\
X_{(n+2)} & AX_{(n+2)} = X_{(n+1)} \\
X_{(n+1)} & AX_{(n+1)} = X_{(n)}
\end{array}
$$

We observe that

$$
\begin{aligned}
X_{(n)} &= AX_{(n+1)} \\
&= A^2 X_{(n+2)} \\
&\ldots\ldots\ldots\ldots \\
&= A^k X_{(n+k)}
\end{aligned}
$$

We notice that in retrospective[4], that if at a certain time $t = n + k$ in the future we want to meet certain demand and the economy is to be producing the output vector $X_{(n+k)}$ then at a time $t = n$ we must have a vector of resources which tends to be the right eigen-vector which can be obtained by a Krilov sequence which we know must converge to the positive eigen-vector corresponding to its Forbenius root λ which for productive economies must be less than one indicating that the input vector at time $t = n$ is obtained as a contraction of the output vector in the future $t = n + k$.

It is interesting to observe that in order to reach any desired level of output in the future through a sequence of input/output transformations of the closed economy, at a certain point in the past the departing vector of resources must be an eigen-vector which can be named an eigen-resource vector.

[4]This argument is similar to the one in Gale D. (1960) regarding a reordering sequence in a closed linear production model

8. **Balanced Trad**

So far we have shown that the eigen-price state of the economy is reached as a result of a natural feedback process where prices affects costs and vice versa. Also we observed that the resources required through time in order to meet a certain level of production must at certain point in time be an eigen-resource vector. Suppose now that our closed economy has reached a state where the output, and the resources are the right eigen-vector and the prices are left eigen-vectors, that is:

$$AX_{(n+1)} = X_{(n)} = \lambda X_{(n+1)}, \textit{and}$$

$$Y^T_{(n+1)}A = Y^T_{(n)} = \lambda Y^T_{(n+1)}$$

The total cost of the inputs to produce $X_{j\,(n+1)}$ units of product *j* is given by

$$C\left(X_{j(n+1)}\right) = X_{j(n+1)} \sum_{i=1}^{n} a_{ij} Y_{i(n)}$$

$$= \quad \lambda X_{j\,(n+1)}\, Y_{i(n)}$$

On the other hand, the total value of the output of product *j* used by the other productive activities is given by

$$\textit{Value of outputs} \quad = \quad Y_{j(n)} * X_{j\,(n)}$$

since $X_{j\,(n)} = \lambda X_{j\,(n+1)}.$,

$$\textit{Value of outputs} \quad = \lambda X_{j\,(n+1)} * Y_{i(n)},$$

Which is the same as cost of inputs above.

This shows that at a certain time $t = n$, if the economy has reached an eigen-price vector $Y_{(n)}$, then at those prices the total cost of the inputs form other production unit required to produce the eigen-output $X_{j\,(n+1)}$ is the same as the value of the quantities of product *j* required by the other production units to produce their respective eigen-outputs.

9. Closed economies eigen-state behavior

Suppose that the economy has reached an eigen-state. That means that it is operating with eigen-prices and eigen-resources so that

$$AX_{(*)} = \lambda X_{(*)},$$

and

$$Y^T_{(*)}A = \lambda Y^T_{(*)}$$

then, the dual pair of linear programming problems corresponding to the closed economy:

$$\begin{array}{ll} \text{Max } C^TX & \text{Min } B^TY \\ \text{s.t. } \quad AX \leq B & \text{s.t. } A^TY \geq C \\ \quad X \geq 0 & \quad Y \geq 0 \end{array}$$

have the property that the eigen-production programs and eigen-prices constitute a pair of optimal solutions respectively. This can be readily seen if we let

$$B = \lambda X_{(*)}$$

and

$$C^T = \lambda Y^T_{(*)}$$

Then it is clear that

$$C^T X_{(*)} = \lambda Y^T_{(*)} X_{(*)} = B^T Y_{(*)}$$

Te above equality holds for feasible $X_{(*)}$ and $Y_{(*)}$ if and only if they are optimal solutions to the respective primal and dual problems.

That is, it follows from the duality theorem of linear programming that the eigen-vectors $X_{(*)}$ and $Y^T_{(*)}$ are optimal solutions to the corresponding linear programming problems of running the economy with a production program that gives an optimal creation of value and with prices that minimize the cost of resources.

Such state where the value of output for one activity (supply) equals the cost of inputs required by that activity (demand), as shown in numeral **9.** above, and is the consequence of the maximizing behavior of self-interest economic agents and in addition produces a global optimum is a very special type of general equilibrium which is denoted here as the ***eigen-state*** of the system or eigen-state of our closed economy.

10. Conclusions

Heinz von Foerster´s observation that social phenomena should be regarded as a closed system whose behavior converges to a certain stability has been successfully established in this paper for the case of closed economies. In fact we have arrived at a general equilibrium for the closed economy by a sequence of operationally closed feedback processes. These results apply to other type of closed systems as the case of systems described by stochastic matrices where the steady state of Markov chains is clearly an eigen-behavior of the system, in fact, stochastic matrices can be considered as a especial case of closed economies where the rate of growth is zero. The fact that eigen-prices here are the consequence of a "natural" formation process, which is not the case in other approaches like in Walras's tatonnement, may explain the worldwide historical stability of certain commodity prices. These results clearly have a major theoretical implication in terms of the role played by the supply and demand on the formation of prices. The eigen-prices are the prices which follow from a state of general equilibrium and conversely. The effect of markets in the formation of prices then must have a secondary importance and account for their fluctuation rather than their formation. The question regarding value as to weather it is determined by the cost of production or by the relative utility to market participants appear to weight more on the side of the cost seen from the feedback process where, even though it may seem a paradox, as in Walras's conception " In general equilibrium every thing depends upon everything else".

References

Aruka, Y., (1990), Perturbation Theorems on the Linear Production Model and Some Properties of Eigenprices, Working Paper in Economics and Econometrics, No. 203, Australian National University

Dietzenbacher (1990) Seton's eigenprices: Further evidence. *Economic Systems Research*, 2 (1990), 103-123.

Dietzenbacher, E. and Wagener, H-J. (1999) Prices in the two Germanies. *Jornal of Comparative Economics* 27, 131-149.

Dorfman, R., Samuelson, P and Solow, R. (1958) "Linear Programming and Economic Analysis", McGraw Hill, New York.

Gale, D. (1960) "The Theory of Linear Economic Models" Mc Graw-Hill, New York

Goodwin, R (1983) Essays in Linear Economic Structures, Macmillan Press, London

Nikaido, H.(1968) "Convex Structures and Economic Theory" Academic Press, New York.

Piaget J. (1975)"Epistemologie Genetique et Equilibration". B. Inhelder, et al (Eds). Delachaux et Niestel.

Piaget, J. (1985). "Equilibration of cognitive structures". University of Chicago Press.

Porter, Michael E. (1985) *Competitive Advantage.* The Free Press, New York

Rocha, L.M.(1998). *Eigenbehavior and Symbols*, *Systems Research* Vol.13 No.3 pp.371-384

Seton, F.,(1989) Eigenprices revisited, Recapitulation and Apologia, In *Economic Systems Research*

Sraffa, P. (1960) "The Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory", Cambridge University Press, Ca

mbridge

Takayama, A.(1974) "Mathematical Economics" The Dryden Presss, Hinsdale.

Varela, F. J.(1979) “Principles of biological autonomy.” New York: North Holland

Von Foerster, Heinz (1977). Objects: tokens for (eigen-) behaviors; ASC Cybernetics Forum 8: pp. 91-96; ed: Sicht und Einsicht. Braunschweig 1985.